International Journal of Theoretical Physics, Vol. 27, No. 12, 1988

Lagrangians for Conformal Gauge Gravity and Conformal Simple Supergravity

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Received June 4, 1988

The connection, curvature, and Lagrangian for a conformal gauge gravity are obtained. A set of generators of the conformal simple supergroup is given, the commutation and anticommutation relations for the superalgebra are calculated, and a Lagrangian of the simple supergravity is established.

1. INTRODUCTION

The Poincaré group $ISO(3, 1)$ is a space-time transformation group often used in the gauge theory of gravity, leaving invariant the metric of space-time; the de Sitter group is a transformation group of space-time with constant curvature, leaving invariant the de Sitter curvature of space-time; and the conformal group $SO(4, 2)$ is also a space-time transformation group, leaving invariant the proper time of space-time. The group $ISO(3, 1)$ is a degenerate version of the de Sitter group, and the de Sitter group is a subgroup of group $SO(4, 2)$, so that the de Sitter gauge theory of gravity will be a subgravity of the conformal gauge theory of gravity, and the $ISO(3, 1)$ gauge theory of gravity (Changgui and Bangquing, 1986) will be a degenerate form of the de Sitter gauge theory of gravity (S. Changgui *et al.*, unpublished). In this paper we construct a Lagrangian for the conformal gauge theory of gravity, give a set of generators for a simple conformal superalgebra, and obtain a Lagrangian for the simple conformal supergravity.

2. LAGRANGIAN OF CONFORMAL GAUGE GRAVITY

Let $\{\partial_\mu\}$ (in this paper indices $\mu, \nu, \ldots = 1, 2, 3, 0$) be a natural basis on the space-time manifold M; then $\forall$point $X \in M$, X^μ are the coordinates

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0020-7748/88/1200-1499$06.00/0

of X. We suppose that the space-time manifold M possesses local conformal invariance; then, for any space-time point X, the local gauge actions of gauge group $SO(4, 2)$ on the manifold M may be expressed by a gauge action space $E_{x(4,2)}$, which is a six-dimensional tangent space of M at X. Let $\{Z_\alpha\}$ (in this paper indices $\alpha, \beta, \ldots = 1, 2, 3, 0, 4, 5$) be a frame in $E_{x(4,2)}$ at X; then all actions exerted by elements of $SO(4, 2)$ at X shall be isomorphic to all actions exerted by the element of $SO(4, 2)$ on the frame at X; thus, we can obtain a set of frames under the actions of $SO(4, 2)$, and denote the set as $\{Z_\alpha\}_x$. Now let

$$P_x = \{Z_\alpha\}_x = \Pi^{-1}(X)$$

where Π is the bundle projection, and take the union $P(M)$ of the P_x at all $X \in M$. Then we have

$$P(M) \equiv \bigcup_{x \in M} P_x(M) = \bigcup_{x \in M} \{Z_\alpha\}_x$$

The bundle $P(M)$ is a frame fiber bundle and it is a principal fiber bundle of which the group $SO(4, 2)$ is the structure group and the space-time manifold M is the base manifold. We can write the principal bundle as

$$P(M) = P(M, SO(4, 2))$$

We have noted that the conformal group $SO(4, 2)$ is a space-time transformation group, which leaves the proper time of space-time invariant, but it is not a linear transformation group of space-time, because the Weyl transformation would not leave invariant the metric of space-time. When we use the above bundle $P(M)$ to describe the gauge actions of group $SO(4, 2)$, we may express the conformal gauge theory of gravity with a linear method.

Let the metric of pseudo-Euclidean space $E_{(4,2)}$ be

$$\eta_{\alpha\beta} = \mathrm{diag}(1, 1, 1, -1, I, -I)$$

We have $I = |\lambda|/\lambda$, where λ is the de Sitter curvature of M. Let the operator generators of group $SO(4, 2)$ be

$$X_{\alpha\beta} = \xi_\alpha \partial_\beta - \xi_\beta \partial_\alpha$$

where ξ_α is a vector in $E_{(4,2)}$. Since the group $SO(3, 1)$ is a subgroup of $SO(4, 2)$, if the generators of $SO(3, 1)$ are M_{ab} (in this paper the indices $a, b, \ldots = 1, 2, 3, 4$), then $M_{ab} = X_{ab}$. We denote generators of de Sitter rotations, the generators of de Sitter boosts, and the dilation generator by P_a, K_a and D, respectively. Then we have

$$P_a = X_{5a} \frac{\sqrt{|\lambda|}}{\varepsilon}, \qquad K_a = X_{6a} \frac{\sqrt{|\lambda|}}{\varepsilon}, \qquad D = -X_{56}$$

where ε is a dimensional constant, $[\varepsilon]=L^{-1}$, and its value is taken as 1. Thus, the commutation relations of generators of group $SO(4,2)$ are

$$\begin{aligned}
&[M_{ab}, M_{cd}] = \eta_{bc}M_{ad} + \eta_{ad}M_{bc} - \eta_{bd}M_{ac} - \eta_{ac}M_{bd}\\
&[M_{ab}, P_c] = \eta_{bc}P_a - \eta_{ac}P_b, \qquad [M_{ab}, K_c] = \eta_{bc}K_a - \eta_{ac}K_b\\
&[P_a, P_b] = -I\lambda M_{ab}, \qquad [K_a, K_b] = I\lambda M_{ab}\\
&[P_a, K_b] = \lambda\eta_{ab}D, \qquad [P_a, D] = -IK_a, \qquad [K_a, D] = -IP_a
\end{aligned}$$

Taking a suitable frame on conformal frame bundle $P(M)$, we may obtain the conformal connection (S. Changgui *et al.*, unpublished)

$$[\mathscr{B}^{\alpha}_{\mu\beta}] = \begin{bmatrix} B^{a}_{\mu b} & V^{a}_{\mu} & C^{a}_{\mu} \\ -IV_{\mu b} & 0 & h_{\mu} \\ IC_{\mu b} & h_{\mu} & 0 \end{bmatrix}$$

where V^{a}_{μ} are Lorentz frame coefficients.

Now we define the horizontal lifting basis

$$\mathscr{D}_{\mu} = \partial_{\mu} + \tfrac{1}{2}\beta^{\alpha\beta}_{\mu}X_{\alpha\beta}$$

Then, by using

$$[\mathscr{D}_{\mu}, \mathscr{D}_{\nu}] = \tfrac{1}{2}\mathscr{R}^{\alpha\beta}_{\mu\nu}X_{\alpha\beta}$$

we may obtain the curvature tensor on $P(M)$ as

$$\mathscr{R}^{\alpha}_{\mu\nu\beta} = \partial_{\mu}\mathscr{B}^{\alpha}_{\nu\beta} + \mathscr{B}^{\alpha}_{\mu\gamma}\mathscr{B}^{\gamma}_{\nu\beta} - (\mu \leftrightarrow \nu)$$

$\mathscr{R}^{\alpha}_{\mu\nu\beta}$ is valued on the Lie algebra $so(4,2)$ and it may be written as the matrix

$$[\mathscr{R}^{\alpha}_{\mu\nu\beta}] = \begin{bmatrix} F^{a}_{\mu\nu b} - IV^{a}_{\mu\nu b} + IC^{a}_{\mu\nu b} & J^{a}_{\mu} + C^{a}_{\mu\nu} & K^{a}_{\mu\nu} + V^{a}_{\mu\nu} \\ -IJ_{\mu\nu b} - IC_{\mu\nu b} & 0 & \partial_{\mu\nu} + IC_{\mu\nu} \\ IK_{\mu\nu b} + IV_{\mu\nu b} & \partial_{\mu\nu} + IC_{\mu\nu} & 0 \end{bmatrix}$$

where

$$\begin{aligned}
F^{a}_{\mu\nu b} &= \partial_{\mu}B^{a}_{\nu b} + B^{a}_{\mu c}B^{c}_{\nu b} - (\mu \leftrightarrow \nu)\\
V^{a}_{\mu\nu b} &= V^{a}_{\mu}V_{\nu b} - C^{a}_{\nu}V_{\mu b}\\
C^{a}_{\mu\nu b} &= C^{a}_{\mu}C_{\nu b} - C^{a}_{\nu}C_{\mu b}\\
J^{a}_{\mu\nu} &= \partial_{\mu}V^{a}_{\nu} + B^{a}_{\mu b}V^{b}_{\nu} - (\mu \leftrightarrow \nu)\\
K^{a}_{\mu\nu} &= \partial_{\mu}C^{a}_{\nu} + B^{a}_{\mu b}C^{b}_{\nu} - (\mu \leftrightarrow \nu)\\
V^{a}_{\mu\nu} &= V^{a}_{\mu}\partial_{\nu} - V^{a}_{\nu}\partial_{\mu}\\
C^{a}_{\mu\nu} &= C^{a}_{\mu}\partial_{\nu} - C^{a}_{\nu}\partial_{\mu}\\
h_{\mu\nu} &= \partial_{\mu}h_{\nu} - \partial_{\nu}h_{\mu}\\
C_{\mu\nu} &= C^{a}_{\mu}V_{\nu a} - C^{a}_{\nu}V_{\mu a}
\end{aligned}$$

We choose the Yang-Mills type of action for this $SO(4,2)$ gauge gravity; then the Lagrangian of the gravity is

$$\mathcal{L}=\tfrac{1}{4}\mathrm{tr}(\mathcal{R}_{\mu\nu}R^{\mu\nu})=-\tfrac{1}{8}\mathcal{R}^{\alpha}_{\mu\nu\beta}\mathcal{R}^{\mu\nu\beta}_{\alpha}$$

Using the above expression, we have

$$\begin{aligned}\mathcal{L}=\tfrac{1}{2}(&-\tfrac{1}{4}F^{b}_{\mu\nu a}F^{\mu\nu a}_{b}-\tfrac{1}{4}V^{b}_{\mu\nu a}V^{\mu\nu a}_{b}+\tfrac{1}{2}IJ^{a}_{\mu\nu}J^{\mu\nu}_{a}\\ &+\tfrac{1}{2}IF^{b}_{\mu\nu a}V^{\mu\nu a}_{b}-\tfrac{1}{2}IF^{a}_{\mu\nu b}C^{\mu\nu b}_{a}+IJ^{a}_{\mu\nu}C^{\mu\nu}_{a}\\ &+\tfrac{1}{2}V^{\mu\nu a}_{b}V^{a}_{\mu\nu b}-IK^{a}_{\mu\nu}V^{\mu\nu}_{a}-\tfrac{1}{4}C^{b}_{\mu\nu a}C^{\mu\nu a}_{b}\\ &+\tfrac{1}{2}IC^{a}_{\mu\nu}C^{\mu\nu}_{a}-\tfrac{1}{2}C_{\mu\nu}C^{\mu\nu}-IC_{\mu\nu}h^{\mu\nu}-\tfrac{1}{2}IK^{a}_{\mu\nu}K^{\mu\nu}_{a}\\ &-\tfrac{1}{2}IV^{a}_{\mu\nu}V^{\mu\nu}_{a}-\tfrac{1}{2}h_{\mu\nu}h^{\mu\nu})\end{aligned}$$

We denote the first four terms by $\mathcal{L}_{\mathrm{ds}}$. Then

$$L_{\mathrm{ds}}=\tfrac{1}{2}(-\tfrac{1}{4}F^{b}_{\mu\nu a}F^{\mu\nu a}_{b}-\tfrac{1}{4}V^{b}_{\mu\nu a}V^{\mu\nu a}_{b}+\tfrac{1}{2}IJ^{a}_{\mu\nu}J^{\mu\nu}_{a}+\tfrac{1}{2}IF^{b}_{\mu\nu a}V^{\mu\nu a}_{b})$$

and $\mathcal{L}_{\mathrm{ds}}$ is the Lagrangian of the de Sitter gauge theory of gravity (S. Changgui *et al.*, unpublished). The four terms of $\mathcal{L}_{\mathrm{ds}}$ are the Einstein, cosmological, torsion, and Einstein-Cartan terms of gravity, respectively. The action of the conformal gauge gravity may be constructed as

$$S=\int\sqrt{-g}\,d^4x$$

where $\sqrt{-g}=V\equiv\det(V^{a}_{\mu})$.

Since there is an Einstein term for gravitation in the conformal gauge gravity but not in the Weyl gravity, in this $SO(4,2)$ gauge theory we may avoid the difficulty we have met in the Weyl gravity. The de Sitter Lagrangian $\mathcal{L}_{\mathrm{ds}}$ is involved in the conformal gauge Lagrangian $\mathcal{L}$, so the de Sitter effect will also appear in the $SO(4,2)$ gauge theory, and GR will be a degenerate case of this gravity. Since we know that the action of the usual Yang-Mills fields is invariant under the Weyl mapping (Yang, 1977), then the Lagrangian $\mathcal{L}$ of the theory will be invariant under the mapping, so this gravity is a conformal gauge gravity holding the Weyl invariance.

3. LAGRANGIAN OF CONFORMAL SIMPLE SUPERGRAVITY

First let us construct a supergroup $SU(2,2|1)$. The usual matrix generators belonging to the subgroup $SU(2,2)$ of supergroup $SU(2,2|1)$ may be written, using Dirac matrices, as

$$M_{ab}=\tfrac{1}{4}[\gamma_a,\gamma_b]=\sigma_{ab}$$

$$P_a=\tfrac{1}{2}\gamma_a,\qquad K_a=\tfrac{1}{2}\gamma_a\gamma_5,\qquad D=\tfrac{1}{2}\gamma_5$$

The above generators are given by the 4×4 matrices, but in the $SU(2,2|1)$ supergravity they must be written as 5×5 matrix forms. When we do this, we may write these generators as 5×5 matrices. However, the elements located at the fifth rows and the fifth columns are all zero. Here we have

$$\{\gamma_a, \gamma_b\} = 2\eta_{ab}, \qquad \gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4, \qquad \gamma_4 = i\gamma_0$$

The generator belonging to the subgroup $U(1)$ of supergroup $SU(2,2|1)$ is the central charge

$$A = -(i/4)\left[\begin{array}{cccc:c} 1 & & 0 & & \\ 0 & 1 & & & 0 \\ & & 1 & & \\ & & & 1 & \\ \hdashline & 0 & & & 4 \end{array}\right]$$

Other generators of $SU(2,2|1)$ are the Majorana spinor charges; we take them as

$$S_\tau = \left[\begin{array}{ccc:c} & & & L_{1a} \\ & 0 & & \vdots \\ & & & L_{4a} \\ \hdashline (CR)_{a1} & \cdots & (CR)_{a4} & 0 \end{array}\right], \qquad Q_\sigma = \left[\begin{array}{ccc:c} & & & R_{1a} \\ & 0 & & \vdots \\ & & & R_{4a} \\ \hdashline (CL)_{a1} & \cdots & (CL)_{a4} & 0 \end{array}\right]$$

where $L = \frac{1}{2}(1-\gamma_5)$ and $R = \frac{1}{2}(1+\gamma_5)$ are chirality projection operators, $C = -C^{-1} = -C^T$ is the charge conjugate matrix, $C = \gamma_0$, $C\gamma_\sigma C^{-1} = -\gamma_\sigma^T$, and in this paper the indices $\sigma, \tau = 1, 2, 3, 4$.

The superalgebra relations of the above 28 generators are

$$[S, M_{ab}] = -(\sigma_{ab}^T)S, \qquad [Q, M_{ab}] = -(\sigma_{ab}^T)Q$$
$$[S, A] = -\tfrac{3}{4}i\gamma_5 S, \qquad [Q, A] = \tfrac{3}{4}i\gamma_5 Q$$
$$[S, D] = -\tfrac{1}{2}S, \qquad [Q, D] = \tfrac{1}{2}Q$$
$$[S, P_a] = -\tfrac{1}{2}\gamma_a^T Q, \qquad [Q, P_a] - \tfrac{1}{2}\gamma_a^T S$$
$$[S, K]_a = -\tfrac{1}{2}\gamma_a Q, \qquad [Q, K_a] = -\tfrac{1}{2}\gamma_a^T S$$
$$\{S_\sigma, S_\tau\} = -\tfrac{1}{2}(\gamma^a C)_{\sigma\tau}(P_a + K_a)$$
$$\{Q_\sigma, Q_\tau\} = \tfrac{1}{2}(\gamma^a C)_{\sigma\tau}(P_a - K_a)$$
$$\{Q_\sigma, S_\tau\} = -\tfrac{1}{2}C_{\sigma\tau}D - (C\sigma^{ab})_{\sigma\tau}M_{ab} + (i\gamma_5 C)_{\sigma\tau}A$$

and

$$[M_{ab}, M_{cd}] = \eta_{bc}M_{ad} + \eta_{ad}M_{bc} - \eta_{bd}M_{ac} - \eta_{ac}M_{bd}$$
$$[M_{ab}, P_c] = \eta_{bc}P_a - \eta_{ac}P_b$$
$$[M_{ab}, K_c] = \eta_{bc}K_a - \eta_{ac}K_b$$
$$[P_a, P_b] = -IM_{ab}, \qquad [K_a, K_b] = IM_{ab}$$
$$[P_a, K_b] = \eta_{ab}D, \qquad [P_a, D] = -IK_a$$
$$[K_a, D] = -IP_a$$

where we let $\lambda = 1$.

If we take supergroup $SU(2,2|1)$ and space-time M as the structure group and base manifold, respectively, we may set up a principal fiber bundle (Kobayashi and Nomizu, 1963) $\hat{P}(M)=\hat{P}(M, SU(2,2|1))$. Let $\hat{\mathscr{B}}_\mu^A$ (in this paper the indices $A, B, \ldots = 1, 2, \ldots, 24$) be the connection on bundle $\hat{P}(M)$; then

$$\hat{\mathscr{D}}_\mu = \partial_\mu + \hat{\mathscr{B}}_\mu^A \hat{X}_A$$

is the horizontal lifting basis, where

$$\hat{X}_A = \{M_{ab}, P_a, K_a, D; S_\sigma, Q_\tau; A\}$$

are the generators of $SU(2,2|1)$. The $\hat{X}_A$ will be a basis on the vertical subspace of bundle space $\hat{P}$, so $(\hat{\mathscr{D}}_\mu, \hat{X}_A)$ may be a basis on the bundle space $\hat{P}(M)$, and

$$[\hat{\mathscr{D}}_\mu, \hat{X}_A]=0$$

For the bundle $\hat{P}(M)$, we may obtain the connection $\hat{\mathscr{B}}_\mu^A$ as

$$\hat{\mathscr{B}}_\mu^A = \{B_\mu^{ab}, V_\mu^a, C_\mu^a, h_\mu; \bar{\phi}_\mu^\sigma, \bar{\psi}_\mu^\tau, A_\mu\}$$

and the curvature $\hat{\mathscr{R}}_{\mu\nu}^A$ as

$$\hat{\mathscr{R}}_{\mu\nu}^A = \partial_\mu \hat{\mathscr{B}}_\nu^A - \partial_\nu \hat{\mathscr{B}}_\mu^A + \hat{\mathscr{F}}_{BC}^A \mathscr{B}_\mu^B \mathscr{B}_\nu^C$$

where $\hat{\mathscr{F}}_{BC}^A$ are structure constants of gauge group $SO(2,2|1)$, and we also have

$$[\hat{X}_A, \hat{X}_B] = \hat{X}_A\hat{X}_B - (-1)^{\hat{\sigma}_A\hat{\sigma}_B}\hat{X}_B\hat{X}_A$$

where $\hat{\sigma}_A, \hat{\sigma}_B$ are the Grassman parity symbols.

The $\hat{\mathscr{R}}_{\mu\nu}^A$ components corresponding to different generators are

$$\hat{\mathscr{R}}_{\mu\nu}^{ab}(M) = F_{\mu\nu}^{ab} - IV_{\mu\nu}^{ab} + IC_{\mu\nu}^{ab} - [\bar{\psi}_\mu(C\sigma^{ab}C^{-1})\phi_\nu + \psi \leftrightarrow \phi]$$

$$\hat{\mathscr{R}}_{\mu\nu}^a(P) = -J_{\mu\nu}^a - IC_{\mu\nu}^a - \tfrac{1}{2}(\bar{\phi}_\mu\gamma^a\phi_\nu + \bar{\psi}_\mu + \gamma^a\psi_\nu)$$

$$\hat{\mathscr{R}}_{\mu\nu}^a(K) = IK_{\mu\nu}^a + V_{\mu\nu}^a - \tfrac{1}{2}(\bar{\psi}_\mu\gamma^a\psi_\nu - \bar{\phi}_\mu\gamma^a\phi_\nu)$$

$$\hat{\mathscr{R}}_{\mu\nu}(D) = h_{\mu\nu} + IC_{\mu\nu} - \tfrac{1}{2}(\bar{\psi}_\mu\phi_\nu + \psi \leftrightarrow \phi)$$

$$\hat{\mathscr{R}}_{\mu\nu}(A) = A_{\mu\nu} + i(\bar{\psi}_\mu\gamma_5\phi_\nu + \psi \leftrightarrow \phi)$$

$$\hat{\mathscr{R}}_{\mu\nu}(S) = \bar{\phi}_\nu\overleftarrow{D}_\mu^1 - \bar{\phi}_\mu\overleftarrow{D}_\nu^1 - \tfrac{1}{2}(\bar{\psi}_\mu\gamma_\nu - \bar{\psi}_\nu\gamma_\mu) - \tfrac{1}{2}(\bar{\psi}_\mu\tilde{\gamma}_\nu - \bar{\psi}_\nu\tilde{\gamma}_\mu)$$

$$\hat{\mathscr{R}}_{\mu\nu}(Q) = \bar{\psi}_\nu\overleftarrow{D}_\mu^2 - \bar{\psi}_\mu\overleftarrow{D}_\nu^2 - \tfrac{1}{2}(\bar{\phi}_\mu\gamma_\nu - \bar{\phi}_\nu\gamma_\mu) + \tfrac{1}{2}(\bar{\phi}_\mu\tilde{\gamma}_\nu - \bar{\phi}_\nu\tilde{\gamma}_\mu)$$

where

$$D_\mu^1 \equiv \partial_\mu + B_{\mu ab}(\sigma^{ab})^T + \tfrac{3}{4}i\gamma_5 A_\mu - \tfrac{1}{2}h_\mu$$
$$D_\mu^2 \equiv \partial_\mu + B_{\mu ab}(\sigma^{ab})^T - \tfrac{3}{4}i\gamma_5 A_\mu + \tfrac{1}{2}h_\mu$$
$$A_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad \gamma_\mu \equiv (\gamma^a)^T V_{\mu a}, \qquad \tilde{\gamma}_\mu \equiv (\gamma^a)^T C_{\mu a}$$

By lifting and lowering the tensor indices, we may obtain the Lagrangian of Yang-Mills type as

$$\hat{\mathscr{L}} = -\tfrac{1}{4}\,\mathrm{tr}(\hat{\mathscr{R}}_{\mu\nu}\hat{\mathscr{R}}^{\mu\nu})$$

and if we write it with component forms corresponding to every generator, $\hat{\mathscr{L}}$ becomes

$$\begin{aligned}\hat{\mathscr{L}} = -\tfrac{1}{4}[&\hat{\mathscr{R}}^{ab}_{\mu\nu}(M)\hat{\mathscr{R}}^{\mu\nu}_{ab}(M) + \hat{\mathscr{R}}^{a}_{\mu\nu}(P)\hat{\mathscr{R}}^{\mu\nu}_{a}(P)\\ &+ \hat{\mathscr{R}}^{a}_{\mu\nu}(K)\hat{\mathscr{R}}^{\mu\nu}_{a}(K) + \hat{\mathscr{R}}_{\mu\nu}(D)\hat{\mathscr{R}}^{\mu\nu}(D)\\ &+ \hat{\mathscr{R}}_{\mu\nu}(A)\hat{\mathscr{R}}^{\mu\nu}(A) + \hat{\mathscr{R}}_{\mu\nu}(Q)C\hat{\mathscr{R}}^{\mu\nu}(S)\\ &+ \hat{\mathscr{R}}_{\mu\nu}(S)C\hat{\mathscr{R}}^{\mu\nu}(Q)], \qquad a > b\end{aligned}$$

Then the action evaluated on the conformal superalgebra $su(2, 2|1)$ is

$$S = -\tfrac{1}{4}\int d^4x\,\sqrt{-g}\,\mathrm{tr}(\hat{\mathscr{R}}_{\mu\nu}\hat{\mathscr{R}}^{\mu\nu})$$

From the Lagrangian $\hat{\mathscr{L}}$ we may see that the graviton and gravitino will be contained in the theory. If the space-time manifold M is flat and only the free field ψ^σ_μ is discussed, we can obtain a spin-3/2 massless gravitino, which satisfies the Rarita-Schwinger equation.

With the generators S_σ and Q_τ we can expand the group $SU(2, 2|1)$ to the case $N > 1$ and establish an expanded supergroup $SU(2, 2|N)$, which may be used to construct the extended conformal supergravity (Changgui *et al.*, 1986).

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